

Section 2 — Fibonacci / ϕ / ψ Mathematics

This section develops the full mathematical foundation of Fibonacci recursion, ϕ (Golden Ratio), ψ (Supergolden Ratio), and their convergence laws as used throughout the FUNt framework.

2.1 Fibonacci Recursive Identity

- $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.
- This recursion forms the basis of all ϕ approximations used in FUNt.
- [Insert visual diagram here]

2.2 Binet Form and ϕ -Dominance

- $F_n = (\phi^n - \psi^n) / \sqrt{5}$.
- For large n , $\psi^n \rightarrow 0$, so $F_n \approx \phi^n / \sqrt{5}$.
- This explains why ϕ dominates harmonic recursion.
- [Insert visual diagram here]

2.3 Convergence Law

- $|F_{n+1} / F_n - \phi| \approx \phi^{-2n}$.
- Error collapse at $n = 10$: $\sim 10^{-4}$.
- Error collapse at $n = 20$: $\sim 10^{-8}$.
- Error collapse at $n = 30$: $\sim 10^{-12}$.
- By $n = 38-40$: machine precision reached.
- [Insert visual diagram here]

2.4 ϕ^n Identity

- $\phi^n = F_n \phi + F_{n-1}$.
- This identity allows mapping any ϕ -power into Fibonacci structure.
- [Insert visual diagram here]

2.5 ψ -Triangle System

- ψ defined by $\psi^3 = \psi + 1$.
- Provides the secondary harmonic backbone of FUNt.

- $\psi \approx 1.46557123187677$.
- ψ contributes to $FHM(n) = \varphi^n + \psi^n$.
- [Insert visual diagram here]

2.6 Spectral Periodicity of Primes (Q($\sqrt{5}$) Field)

- Prime index cycles follow ≈ 0.356 oscillation.
- $\sqrt{5} / (2\pi)$ resonance visible in FFT of first 10M primes.
- 0.92φ bias across independent physical laws.
- This section unifies number theory and FUNt harmonic physics.
- [Insert visual diagram here]

2.7 ϕ^7 as Central Quantizer

- $\varphi^7 = 29.0344418537$.
- This is the dominant coherence ceiling in FUNt mathematics.
- Ties directly into recursion, decay, bloom, and particle decad math.
- [Insert visual diagram here]